# Numerical Computation of Hydro Electrical Generators Field Form Coefficients

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***Abstract* - The field form factor and Carter's coefficient are the key parameters in the optimization process of large electrical machines. These coefficients are usually computed by analytical methods with many assumptions. In this paper, a numerical approach based on the finite element analysis (FEA) is proposed to compute these parameters achieving better accuracy. The advantage of this procedure is that it allows taking into account the whole harmonic content. The proposed approach is first described and applied to evaluate these two coefficients in case of existing large hydro electrical generator and then compared to that obtained using analytical methods in order to evaluate the accuracy of the latter.**

***Index Terms*— Carter's coefficient, field form factor, hydro electrical generator, magnetic flux density waveform.**

## I. INTRODUCTION

THE DESIGN of a hydro electrical generator is a very difficult task because of its complex geometry. It is well known from the industry that the waveform of the induced voltage, reactance, vibration and noise and some losses are in many cases related to the pole face geometry. Some coefficients are computed analytically and used to optimise these losses. Each analytical method has its own hypotheses and simplifications and the accuracy of the obtained results is mostly dependent on these assumptions; where for example, the presence of time and space harmonics in the air gap are not taken into account in most analytical methods.

The two coefficients involved in this optimization process are mainly the field form and Carter's factors. Unfortunately their definitions are not unique throughout the literature. For instance one can find different definitions for the first factor as in [1] - [3] and [4] - [5] for the second one. Moreover, the main problem regarding the previous definitions is that it is not possible to evaluate all involved variables and then many hypotheses are necessary to compute analytically these coefficients. In [6] - [7], the authors computed Carter factor using FEM to study the effects of the slot fringing in cylindrical rotor and permanent-magnet electrical machines respectively.

In this paper, a new approach based on the finite element analysis is introduced to compute the two previous coefficients and then compare the obtained results to that coming from the analytical methods in order to evaluate the accuracy of the latter.

## II. DEFINITIONS OF THE FIELD FORM COEFFICIENTS

### A. Field form factor

Generally it is assumed that the magnetic flux density in the air gap has a perfect sinusoidal waveform. In fact, the presence of harmonics du to the shape of the pole and then the variation of the air gap gives a particular waveform to the magnetic flux density in the air gap [2]. The field form factor ($k_\phi$) was introduced to take into account the effect of the pole shape and its definitions given in [1], [2] and [3] are respectively that of (1), (2) and (3).

$$k_\phi = \frac{\phi_r}{\phi_0} \tag{1}$$

$$k_\phi = \frac{B_{r,max}}{B_{1,max}} \tag{2}$$

$$k_\phi = \frac{B_{1,max}}{\bar{B}_r} \tag{3}$$

Where $\phi_r$ and $\phi_0$ represent the flux per pole and its fundamental component respectively; $B_{r,max}$ , $B_{1,max}$ , $B_{1,max}$and $\bar{B}_r$ are respectively the magnitude, the magnitude of the fundamental component, RMS and mean values of the radial flux density in the air gap.

### B. Carter's Coefficient

Carter's coefficient ($k_c$) allows taking into account the presence of teeth in front of each pole that affects the distribution of the magnetic flux density in the air gap. For this reason, it can be defined as the ratio of the effective length (y') to that of the smooth model without stator slots (y). For an alternator, the length "y" corresponds to the pole pitch as shown by equation (4).

$$k_c = \frac{y}{y - \delta w_s} = \frac{\tau_d}{\tau_d - \delta w_s} \tag{4}$$

Where $w_s$ is the width of the slot and the length gap ($l_e$) is the minimum gap under a pole. The problem is that there are several expressions for "δ" that can be found in the literature as in [4] - [6] where equations (5) – (7) were respectively given:

$$\frac{w_s}{w_s + 5l_e} \tag{5}$$

$$\frac{2}{\pi}\left[ tan^{-1}\left(\frac{w_s}{2l_e}\right) - \frac{l_e}{w_s} log_e\left(1 + \left(\frac{w_s}{2l_e}\right)^2\right)\right] \tag{6}$$

$$\frac{5l_e}{5l_e + w_s} \tag{7}$$

## III. NUMERICAL METHOD APPLICATION TO FIELD FORM COEFFICIENTS COMPUTATION

The numerical simulation is used to compute the field form factor based on the definitions given by (1) – (3) because they all involve the real magnetic flux density in the air gap. The numerical model allows knowing the magnetic flux density without making any assumption. The method used to compute Carter's coefficient is based on the ratio between the numerical magnetic potential and the one obtained with a "smooth" model. Hence, for each analytical value based on the used definition ((1)-(3) for the field form factor and (5)-(7) for Carter's coefficient) a relative error is computed as follows:

$$Error(\%) = 100\frac{(numerical\ value - analytical\ value)}{numerical\ value} \tag{8}$$

Where the analytical value for the field form factor is $B_{r,analytical}$ computed with the help of the geometry of the generator and the numerical value is that obtained with the numerical model. For Carter's coefficient the analytical value is $\theta_{r,analytical}$ and the numerical value comes also from the numerical model.

## IV. NUMERICAL RESULTS

The hydro electrical generator under study has the following specifications: 432 slots, 68 poles and a bore diameter of 7.5 m. The obtained results by numerical simulation are depicted in Fig. 1 and summarized in Table 1.

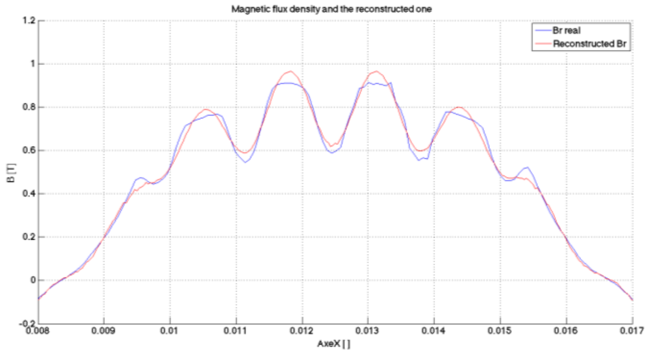

Fig. 1 Magnetic flux density as a function of space (blue) and the reconstructed signal (red)

TABLE I

ANALYTICAL METHODS ACCURACY

| Definition | Field form factor | Error (%) | Definition | Carter's coefficient | Error (%) |
|---|---|---|---|---|---|
| (1) | 1.01 | 7.8 | (5) | 1.135 | 3.58 |
| (2) | 1.086 | 0.92 | (6) | 1.22 | 11.29 |
| (3) | 1.125 | 2.63 | (7) | 1.439 | 31.35 |

Based on the results given in Table 1, one can conclude that (2) and (5) are the best candidates respectively for the field form factor and Carter's factor computation. For the field form factor only (2) and (3) give an acceptable error. It was also noticed that the result heavily depends on the chosen definition of Carter's factor. The error can be as high as 31.35% and one has to be careful in selecting the right definition of this coefficient.

To go further these coefficients should be computed for running hydro electrical generators. The challenge is that a magnetic flux density sensor gives the time distribution and it is necessary to know the space one to compute the field form coefficients. To reconstruct the space distribution from the time one, the idea is to add the frequency component to the variation of the magnetic field due to the teeth. The sensors give the variation of the magnetic flux as it has a definite location which is generally on a tooth and the influence of the slots between teeth is not reflected. It is possible to reconstruct the space distribution simply by adding the spatial frequency of the teeth as shown on Fig. 1. Using the time distribution obtained with sensors placed on four different teeth of the machine, the field form factor is computed and the results are given in Table 2.

TABLE 2

NUMERICAL METHOD ACCURACY

| Definition | Experimental value | Numerical value |
|---|---|---|
| (1) | 1.12 | 1.01 |
| (2) | 1.01 | 1.086 |
| (3) | 1.125 | 1.04 |

## V. CONCLUSION

Generally the companies compute the field form coefficients by means of the analytical approach with many assumptions. Thanks to the numerical software we now can compute them more precisely.

In this paper, the numerical simulation of a hydro electrical generator was performed. Then all necessary signals values were extracted and the results regarding the field form coefficients using the numerical approach were obtained. The designer will obtain a better prediction of the magnetic flux density and the magnetic potential using the coefficients found with the proposed methods. It is also shown that the magnetic flux measurement on an operating machine combined with the numerical method can be used to compute more accurately these coefficients; by reconstructing the space distribution of the magnetic flux density from the time varying signals coming from magnetic flux sensors.

## VI. REFERNCES

[1] F. Bernot, "Alternateur de grande puissance. Partie 1", *Technique de l'Ingénieur*, d3550, pp. 15-24, 2002

[2] J.-D. Chatelain, " Traité d'électricité : Machine électrique, volume X", *Ed. Georgi*, vol. 2,1981

[3] J. Marcel, "Traité d'électricté : Electromécanique, volume IX", *Ed. Presse polytechniques et universitaires romandes*, 2004

[4] Freeman, E. M. "The calculation of harmonics, due to slotting, in the flux-density waveform of a dynamo-electric machine," *Proceedings of the IEE-Part C: Monographs*, vol. 109, no. 16, pp. 581-588, 1962.

[5] F. Bernot, "Alternateur de grande puissance. Partie 2", *Technique de l'Ingénieur*, d3551, pp. 8-11, 2003

[6] M.B.B. Sharifian *et al.*, "Slot fringing effect on the magnetic characteristics of the electrical machines", *ICECS. Proceedings of the 2003 10th IEEE International Conference on Electronics, Circuits and Systems*, vol. 2, pp. 778-781, 2003.

[7] Hung Vu Xuan, D. Lahaye, H. Polinder, J. A. Ferreira, "Influence of stator slotting on the performance of permanent-magnet machines with concentrated windings," *IEEE Transactions on Magnetics*, vol. 49, pp. 929-938, 2013.